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FEEDBACK SYSTEMS AND SIMULTANEOUS DESIGN R. Saeks, Principal Investigator March 1, 1981 - July 15, 1983



Although computer-aided design is widely accepted in the electronics industry, where it is essential to the design of any modern commercial integrated circuit, research on computer-aided control system design has been held back by the primitive electro-mechanical and hydraulic hardware with which control systems have historically been implemented. Indeed, computer aids are not required for the design of the type of low order (usually) single loop control systems which can be implemented on such hardware. With the development of the modern microcomputer, however, it is now feasible to implement a sophisticated high order multi-loop control strategy and, as such, the control community has begun to look towards CAD with renewed interest. 8,9,10,11

Unlike manual design techniques, wherein the designer's intuition can compensate for the lack of a complete analytic theory, a CAD package must to based on a precise analytic theory if it is to be successfully implemented. Indeed, the alternatives of brute force optimization and/or disorganized interactive searches are neither cost effective nor "socially" acceptable to the engineering community. As such, the goal of the research described in the present proposal is the development of an analytic theory which can serve as the basis of a CAD package. In particular, we are in the process of developing a complete analytic theory for the incorporation of the fundamental asymptotic and, hopefully, initial value constraints into a CAD package. Specific research topics will include:

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- (i) pole placement
- (ii) initial value constraints
- (iii) design of time-varying systems
- (iv) simultaneous design

Our theory is predicted on the, now classical, stabilization theory developed during the mid-seventies by Youla, et al. 18,19,20 and its extension into a general purpose asymptotic design theory as formulated by the author, Desoer, Liu, Vidyasagar, Francis, et al. 4,5,11,12,13,14,17 Since the theory is now well known we will simply review the essentials here, formulating the notation which will be required in the sequel. For brevity this review will be restricted to the single variate case though, in fact, most of the theory can be extended to the general linear case.

As usual, we let p(s) denote a single variate rational plant. Here, p(s) may be unstable or even improper. Of course, as with any rational function p(s) can be represented as the ratio of polynomials. For our purposes, however, we find it more convenient to express p(s) as the ratio of stable rational functions, p(s) and p(s).

$$p(s) = n(s)/d(s)$$
 (1.1)

As with the classical polynomial representation we require that n(s) and d(s) be coprime, though in the context of our rational fractional representation it suffices to only require that n(s) and d(s) have no common (closed) right half plane zeros. Indeed, "real world" control system designers often permit left half-plane pole/zero cancellations.

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Interestingly, this coprimeness condition manifests itself in the  $\underline{\mathtt{Bezout}}$  equality

$$u(s)n(s) + v(s)d(s) = 1$$
 (1.2)

i.e., stable rational functions n(s) and d(s) are coprime in the sense that they have no common right half-plane zeros if and only if there exists stable rational functions, u(s) and v(s), such that 1.2 is satisfied.

Given the above notation it can be shown that a compensator, c(s), will stabilize a unity gain feedback systems with plant, p(s), as illustrated in Figure 1 if and only if it takes the form

$$c(s) = \frac{-d(s)w(s) + u(s)}{n(s)w(s) + v(s)}$$
(1.3)

where w(s) is an arbitrary stable rational function (which serves as our design parameter).



Figure 1. Single loop unity gain feedback system.

Moreover, the various <u>feedback systems gains all turn out to be linear in</u>
the design parameters, w(s). For instance, the system input/output gain
takes the form

As such, one can formulate a design theory in which the stability question is totally resolved and at the same time the remaining design problem is simplified by the linearity of the feedback system gain expressions. Indeed by restricting w(s) to stable rational functions of the form

$$w(s) = a(s)\widetilde{w}(s) + b(s)$$

where a(s) and b(s) are appropriately defined stable rational functions and w(s) is a new stable rational design parameter we can incorporate essentially any asymptotic design constraint into our theory. Specific constraints which we have investigated 13,14 include:

- (i) tracking
- (ii) disturbance rejection
- (iii) robust design
- (iv) model matching
- (v) pole placement
- (vi) design with restricted compensators
- (vii) simultaneous design

The goal of the research is to refine the above described results, especially with regard to the <u>pole placement</u>. To extend the theory to permit the inclusion of <u>initial value constraints</u>, and to investigate the possibility of applying the theory to <u>linear time-varying system</u> via the medium of a recently developed generalized frequency domain theory.

During the past year our work has been concentrated on the solution of the single and multivariate pole placement problems 2,6,18 and on the extension of our theory to the case of linear time-varying systems.

SINGLE-VARIATE POLE PLACEMENT: By the pole placement problem  $^{6,14}$  in our frequency domain setting we refer to the problem of finding a stabilizing compensator, c(s), for a given plant, p(s), such that the input/output gain, h(s), for the feedback system of Figure 1 takes the form

$$h(s) = r(s)/q(s)$$
 (2.1)

where q(s) is a prescribed Hurwitz polynomial and r(s) is an arbitrary polynomial such that

$$o(r) \leq o(q) \tag{2.2}$$

Here o() denotes the order of its polynomial argument and equation 2.2 is required to guarantee that h(s) has no poles at infinity. It is interesting to compare this "frequency domain pole placement" problem with the classical state-space pole (actually eigenvalue) placement problem. In the latter case memoryless state feedback is employed, thereby fixing the order of the resultant feedback system, whereas, in our case dynamic feedback is employed allowing the designer to change the degree of the system. Indeed, as we will see, the degree of the resultant system is of primary importance in our theory.

About a year ago we gave what we thought to be a correct solution to the "frequency domain pole placement" problem to the effect that the problem admitted a solution if and (generically) only if

$$o(q) \ge \pi(p) = 1$$
 (2.3)

where  $\pi(p)$  is the total number of (closed) right half-plane poles and zeros of the plant transfer function, p(s), including poles and zeros at infinity. In the process of trying to extend the result to the multivariate case, however, it became apparent that 2.3 was in error by "one" in certain cases. As such, a major endeavor during the past year has been the formulation of a correct pole placement theorem for the single-variant case.

To formulate the pole placement theorem we let p(s) have a rational fractional representation as per equations 1.1 and 1.2 and then further decompose the numerator and denominator functions as

$$n(s) = \underbrace{\tilde{a}(\underline{s})\tilde{a}(\underline{s})}_{m(s)}$$
 (2.3)

and

$$d(s) = \frac{\tilde{b}(s)\tilde{b}(s)}{\tilde{m}(s)}$$
 (2.4)

Here  $\tilde{a}(s)$  and  $\tilde{a}(s)$  are polynomials representing the left and right halfplane zeros of n(s),  $\tilde{b}(s)$  and  $\tilde{b}(s)$  are polynomials representing the left and right half-plane zeros of d(s), and m(s) is a Hurwitz common denominator for n(s) and d(s). Our pole placement theorem then results from the following lemma. 12

LEMMA: The single-variate pole placement problem admits a solution if and only if the polynomial equation

$$a(s)x(s) + b(s)y(s) = q(s)$$
 (2.5)

admits polynomial solutions, x(s) and y(s), such that

$$o(x) < o(q) + o(a) - o(m)$$
 (2.6)

$$o(y) < o(q) + o(b) - o(m)$$
 (2.7)

Moreover, q(s) and r(s) are (polynomial) coprime if and only if x(s) and y(s) are (polynomial) coprime.

To obtain 2.3 from the lemma all that is necessary is to count equations and unknowns in 2.5 through 2.7. In fact, however, the requirement that x(s) and y(s) be polynomial induces additional constraints to the effect that x(s) and y(s) have non-negative order. Surprisingly, these constraints can become active just to the extent that the "1" in equation 2.3 may drop out. Counting equations and unknowns with this additional constraint then yields the following corrected single-variate pole placement theorem. Note that the linear independence of the set of linear equations defined by 2.5 follows from the coprimences of  $\hat{a}(s)$  and  $\hat{b}(s)$ .

THEOREM: The single-variate pole placement problem admits a solution if and (generically) only if

where j is either 1 or 0 as indicated in Table 1.

|          | j = 1                           | j ≈ 0                        |
|----------|---------------------------------|------------------------------|
| p(s)     | $o(m) = o(a) \ge 1$ and         | $o(m) - o(\tilde{a}) = 0$ or |
| proper   | o(b) > 1                        | o(b) = 0                     |
| p(s)     | $o(m) - o(\tilde{b}) \ge 1$ and | $o(m) - o(\tilde{b}) = 0$ or |
| improper | o(b) > 1                        | o(b) = 0                     |

Table 1. Perturbation factor for the single-variate pole placement theorem.

MULTIVARIATE POLE PLACEMENT: As a starting point for our research on the multivariate pole placement problem we have considered a <u>stable</u> rational multivariate plant characterized by an n by n transfer function matrix, P(s). Since P(s) is stable it admits a trivial rational fractional representation with numerator P(s) and denominator equal to the identity. Of course, this representation is trivially (right and left) coprime via the equality

$$[0][P(s)] + [1][1] = 1$$
 (2.9)

and, as such, it is stabilized by compensators of the form  $^{4}$ 

$$O(s) = [W(s)P(s) + 1]^{-1}[W(s)]$$
 (2.10)

where W(s) is an arbitrary stable rational matrix. Moreover, in this special case the input/output gain for the resultant feedback system takes the form

$$H(s) = P(s)W(s)$$
 (2.11)

In this highly specialized multivariate case we formulate the pole placement problem by requiring that W(s) be chosen to stabilize the system and yield an input/output gain of the form

$$H(s) = R(s)/q(s)$$
 (2.12)

where q(s) is a prescribed Hurwitz polynomial and R(s) is an arbitrary n by n polynomial matrix such that

$$o(R) \le o(q) \tag{2.13}$$

where the order of a matrix is defined to be the maximum of the orders of its components.  $^{6}$ 

Given P(s) one may factor out a (Hurwitz) common denominator, b(s), and then transform the numerator into Smith canonical form. After replacing the common denominator this then yields the Smith-MacMillan form for P(s) of 2.14.

$$P(s) = U(s)D(s)V(s)$$
 (2.14)

where U(s) and V(s) are unimodular polynomial matrices and D(s) is a diagonal matrix with (possibly zero) stable rational entries:

$$d_{i}(s) = \tilde{a}_{i}(s)\hat{a}_{i}(s)/\tilde{b}_{i}(s)$$
 (2.15)

Letting  $\mathbf{V_i}(\mathbf{s})$  denote the ith row of  $\mathbf{V}(\mathbf{s})$  we then obtain the following theorem.  $\mathbf{6}$ 

THEOREM: i) If the multivariate pole placement problem admits a solution then

$$o(q) \ge o(\tilde{b}_i) - o(\tilde{a}_i) - o(v_i)$$
 (2.16)

for all  $i = 1, \ldots, n$ .

ii) The multivariate pole placement problem admits a solution if

$$o(q) \ge o(\tilde{b}_i) - o(\tilde{a}_i)$$
 (2.17)

for all  $i = 1, \ldots, n$ .

Unfortunately, the <u>necessary and sufficient conditions of 2.16 and 2.17 do not coincide</u> except in the single-variate case where they coincide with the (corrected) single-variate theorem.

Needless to say the above theorem leaves many unanswered questions. In particular, since the Smith form is not unique the necessary condition of the theorem can conceivably be tightened by simply varying the Smith form employed. Even here, however, it is not clear if a single Smith form will minimize the expression of equation 2.16 simultanes soly for all i

(personally I doubt it). Furthermore, what happens if we formulate a dual necessary condition in terms of the columns of U(s)?

TIME-VARYING SYSTEMS: Although many of the above techniques can be formulated in an abstract operator theoretic setting their power and potential applicability to the CAD problem lies with the simple frequency domain formulation employed. As such, during the past year we have endeavored to formulate a viable frequency domain theory which is applicable to time-varying systems and, yet still preserves to the classical time-invariant frequency domain concept. Of the classical time-invariant frequency domain concept. Of the classical time-invariant frequency domain concept. Of the classical time-invariant frequency domain concepts have been formulated to ever the years (including several by the principal investigator) using functions of several variables, matrix and operator valued functions, etc.

In the present formulation, motivated by the work of Arveson and his students, 1,7 we represent a time-varying system by an operator valued function of frequency\* which, we believe, has every analytic property of the classical frequency response for time-invariant operators, though it does not yield the expected computational simplification except in the time-invariate case. 2,15

In its most general form (a Hilbert resolution space or its generalizations) the required frequency response takes the form:

$$\Lambda(\omega) = V(\omega)^{-1} \Lambda U(\omega) \tag{2.18}$$

<sup>\*</sup>At first it may ream redundant to represent a single operator, the given time-varying rystem, by an operator valued function (whose value at 0, ry the way, scincines with the ariginal operator). In fact, now-ver, what this does is to contine information about the time (or frequency) structure of the underlying space with that about the operator into a single operator valued function which rimultaneously characterizes to the given operator and the space on which it is defined.

where  $U(\omega)$  is the group of "frequency shift" operators. For proving we will simply illustrate the concept in the "real world" continuous time constant where  $U(\omega)$  is, vias classical formule, transform theory, the operator which multiplies a function, f, by  $exp(i\omega t)$ 

$$[U(\omega)f] = \exp(i\omega t)f(t)$$
 (2.19)

Interestingly, for a simple time-varying differential operator, A, given by

$$X' = A(t)X + B(t)u$$

$$y = C(t)X + D(t)u$$
(2.20)

 $A(\omega)$  is given by the surprisingly simple expression

$$X' = [\omega I + A(t)]X + B(t)u$$

$$y = C(t)X + D(t)u$$
(2.21)

Here,  $\omega$  is the parameter and, as such, 2.21 should be interpreted as a differential operator valued function of frequency. Although the simple expression of 2.21 is somewhat surprising if one pertricts (.21 to the time-invariant case we then obtain

$$X' = 1.1 + A3X + Bu$$

$$y \in CX + Du$$
 (7.12)

Now, this yields the classical frequency response valued function of  $\boldsymbol{\omega}$  given by

$$A(\widetilde{\omega}, \omega) = [C(\widetilde{\omega}I - \omega I - A)^{-1} B + D]$$
 (2.23)

As such, even though 2.23 technically has two frequency variables ( $\omega$  is a parameter and  $\tilde{\omega}$  is the variable of a multiplication operator) they are clearly redundant and may be eliminated by the change of variable

$$\hat{\omega} = \hat{\omega} - \omega \tag{2.24}$$

whence we obtain the classical frequency response function

$$\hat{A(\omega)} = [C(\hat{\omega}I - A)^{-1}B + D] \tag{2.25}$$

Although the above formulation does not yield the kind of computational simplification associated with the classical time-invariant frequency response it appears to retain every analytic characteristic usually associated with the classical frequency response. 2,15 As such, we believe that it can serve as the ideal medium with which to extend our control system design concepts to the time-varying case and have undertaken a preliminary investigation of this possibility.

CTALE ACTIVITIES: In addition to the above described rescarch we have ecutinged our investigation of the simultaneous design problem  $^{10,14}$  and initiated a preliminary investigation of the adaptive control problem from

the point of view of our design theory. The former problem has proven to be extremely hard especially in the discrete case. In this endeavor we have completed our <u>abstract algebraic-geometric characterization</u> of the simultaneous stabilization problem which yields a complete but noncomputations solution. Furthermore, we have continued our work on the discrete problem, reducing it to an (albeit unsolvable) <u>interpolation problem</u>. Interestingly, some parallel work by Byrnes, et al. has yielded some generic, but equally untestable results.<sup>3</sup>

In the adaptive control area we have formulated an approach to the problem which yields a solution which is always globally stable but is only locally stable when the model for our plant is sufficiently close to the actual plant. At the present time we are trying to determine "how close" is sufficiently close. To this end we have conjectured that the model must lie in the same connected component (of an appropriate space of plants) as the actual plant. At the present time this work is in a very preliminary form and, indeed, the precise definition of the appropriate "space of plants" is still vague.

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## PERSONNEL

- R. Saeks, Principal Investigator.
- A. Tyer, Research Assistant/Research Associate; received Ph.D. Dec. 1982, working under the grant with a dissertation entitled: "Feedback Systems Design: The Pole Flacement Problem.
- G. Enowles, (Formerly G. Achton), Faculty Accordate.

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- R. Saeks, 1983 IEEE Dec. and Cont. Conf., San Antonio, Dec. 1983.
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- R. Saeks, Briefing to Personnel at AFWL (Col. Stans), Kirkland AFB, Sept. 1982.

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